Monopoly

1 Monopoly

Consider the case of a monopolist whose cost function is $c(q) = q^2$ and whose inverse demand function is given by $\mathrm{p(q) = a - q}$.

1. State the maximization problem and solve it (prices, quantities, profit).
2. Suppose the monopoly can make a one-time payment of K in advertising such that the inverse demand becomes $p(q) = 2a–2q$. Find the maximum value of K that the monopoly is willing to pay.

Solution

1. First, I state the monopolist's profits:

$$\pi = (a - q)q - q^2$$

I obtain the first order condition:

$$\frac{\partial \pi}{\partial q} = a - 2q - 2q = 0$$

$$q = a/4$$

I obtain the price:

$$p = a - a/4 = 3a/4$$

I obtain the profits:

$$\pi = (3a/4)(a/4) - (a/4)^2 = 2a^2/16 = a^2/8$$

2. If the monopolist pays K, the profit would be:

$$\pi = (2a - 2q)q - q^2 - K$$

Maximizing:

$$\frac{\partial \pi}{\partial q} = 2a - 4q - 2q = 0$$

Solving for q:

$$q = a/3$$

We obtain the price:

$$p = 2a - a2/3 = 4a/3$$

Therefore, the profits would be:

$$\pi = (4a/3)(a/3) - (a/3)^2 - K = a^2/3 - K$$

Therefore, the decision to pay will occur when the following happens:

$$a^2/3 - K \geq a^2/8$$

$$\frac{5a^2}{24} \geq K$$

That is, the maximum he is willing to pay is $5a^2/24$.